Statistical multi-scale method of mechanics parameter prediction for rock mass[1]

J.Z. Cui [1,2] & F. Han [1] & Y.J. Shan [2]
[1] *Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, China*
[2] *Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing, China*

ABSTRACT: In this paper a statistical multi-scale method for the mechanics parameter prediction of the rock mass with random distribution of multi-scale cracks/joints is presented. First the micro-structure of the rock mass with random distribution of multi-scale cracks/joints is represented. Then the statistical second-order two-scale method for the mechanics performance predictions of the rock mass structure with random cracks/joints distribution is presented, including the statistical second-order two-scale expression on the vector-valued displacement, strain tensor and stress tensor, and the algorithm procedure of statistical multi-sale computation for the mechanics parameters. Finally some numerical results for mechanical parameters for the rock mass with random distributions of multi-scale joints/cracks by statistical multi-scale method are shown.

1 INTRODUCTION

With the rapid advance of engineering science, especially computing technology, the computational engineering science is developing very fast. A variety of numerical methods for the predicting the physical and mechanical performance of materials was developed in last decade.

According to their micro-structure the composite materials can be divided into two classes: composite materials with periodic configurations (Cui et al. 1997 Cui & Shan 2000) and composite materials with random distribution (Li & Cui 2004). A lot of random composite materials exist in nature and human life，such as rock mass and concrete (Shan et al. 2002). Due to the difference of their micro-configurations it needs to make use of different numerical methods to evaluate the physical and mechanical performance of them.

For the composite materials with random distribution some works have been done for predicting the physics and mechanical properties of random particulate composites (Li & Cui 2005 Yu et al. 2008). Many approaches can be used to the calculation of macroscopic stiffness parameters, such as the law of mixture, Hashin-Shtrikman upper and lower bounds method, self-consistent approach and Eshelby effective inclusion method etc. However, in regard to the prediction for strength parameters there are few theoretical techniques available, and most of them are based on the greatly simplification of real composite structures. Till now there is still no multi-scale analysis method to predict the physical and mechanical performance of the rock mass structure with random joints or/and cracks distribution.

In this paper a new statistical multi-scale method is presented to predict the mechanical performance of rock mass with random joint and/or crack distribution and related structures.

The remainder of this paper is outlined as follows. In section 2 the representation of the rock

[1] Supported by the National Natural Science Foundation of China (10590353 and 90405016)

mass with random distributions of multi-scale joints/cracks briefly described. The section 3 is devoted to the statistical second-order two-scale formulation for the prediction of the materials with random distribution and related structure. In section 4 the algorithm procedure for statistical multi-scale computation of rock mass with random distributions of multi-scale joints/cracks is given. In section 5 some numerical results for mechanical parameters of the rock mass with random distributions of multi-scale joints/cracks are shown.

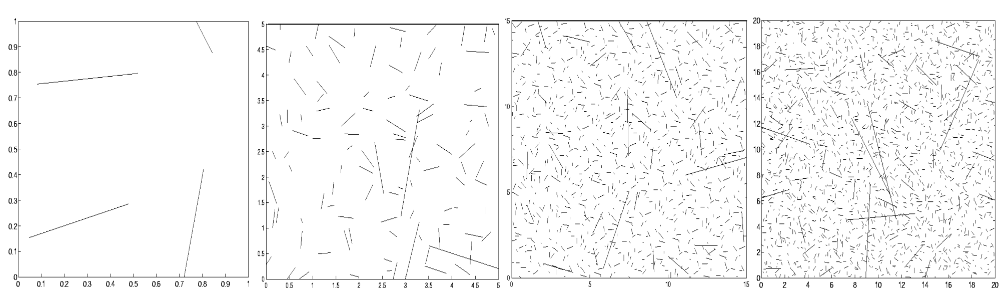
2 MULTI-SCALE REPRESENTATION OF ROCK MASS WITH RANDOM JOINT/CRACK DISTRIBUTION

The materials with random joint/crack distribution, such as rock mass and damaged materials, can be represented as follows: all of the joints inside investigated structure are divided into several groups in their lengths l^j and $\varepsilon^j > l^j > \varepsilon^{j+1}$.

From the survey of engineering geology and the fitting method of statistic data, for each group of joints/cracks the probability distribution of the joints inside structure Ω can be described as follows:

1. The long joints, whose length $l > \alpha L$ and L is the size of structure Ω, are considered determinate, generally, choose $\alpha \approx 10^{-1}$.
2. Choose $\varepsilon^j \left(j = 1, 2, \cdots m \right)$ and $\alpha L > \varepsilon^1 > l^{j-1} > \varepsilon^j > 0$ the statistical model of joints with the length l^j satisfied $\varepsilon^j > l^j > \varepsilon^{j+1}$ can be determined in following way:
   (a) Specify the density of joint distribution and the distribution model of central points of joints, for example, uniform distribution in Ω.
   (b) Specify the distribution model for trace lengths of joint surfaces, for example, normal distribution round the value of some length.
   (c) Specify the distribution model for inclinations of joint surfaces in $[0, \pi/2]$, for example, some normal distribution round some angle. And specify the distribution model for trends of joint surfaces in $[-\pi/2, \pi/2]$, for example, some normal distribution round some angle.
3. For most of rock mass structures there is some jointing inside joint surfaces, and it occupies a certain thickness. So the thickness of jointing must be specified, for example, it is supposed to be a function depended on its trace length.
4. The physical or mechanical parameters of intact rock and jointing must be prescribed.

From previous representation in rock mass structure Ω one can obtain a sample of every group of joints with lengths l^j $(j = 1, 2, \cdots, m)$ where $\varepsilon^j > l^j > \varepsilon^{j+1}$, and then periodically obtain a concrete distributions $\left\{ a_{ij}^{\varepsilon}(x, \omega) \right\}$ and $\left\{ a_{ijhk}^{\varepsilon}(x, \omega) \right\}$ on physical and mechanical parameters on Ω. As example, a sample distribution for a kind of rock mass is shown in Figure 1.



a . Joints in ε^4-screen b. Joints in ε^3-screen c. Joints in ε^2-screen d. Joints in ε^1-screen
Figure 1. The joint statistical model of the four screen scales in rock mass.

3 STATISTICAL SECOND-ORDER TWO-SCALE FORMULATION OF THE STRUCTURE WITH RANDOM JOINT/CRACK DISTRIBUTION

3.1 *Statistical two-scale formulation for the composites with random distribution*

In this section based on the representation previously the structures with random distribution of one scale joints/cracks is investigated, and it has only same ε-size statistic screen. Its elasticity problem with mixed boundary conditions can be expressed as follows:

$$\begin{cases} \dfrac{\partial}{\partial x_j}\left[a^{\varepsilon}_{ijhk}(x,\omega)\dfrac{1}{2}\left(\dfrac{\partial u^{\varepsilon}_h(x,\omega)}{\partial x_k}+\dfrac{\partial u^{\varepsilon}_k(x,\omega)}{\partial x_h}\right)\right]=f_i(x) & x\in\Omega \\ \sigma_i(x,\omega)=\nu^{(1)}_j a^{\varepsilon}_{jihk}(x,\omega)\dfrac{1}{2}\left(\dfrac{\partial u^{\varepsilon}_h(x,\omega)}{\partial x_k}+\dfrac{\partial u^{\varepsilon}_k(x,\omega)}{\partial x_h}\right)=p_i(x) & x\in\Gamma_2 \\ \mathbf{u}^{\varepsilon}(x,\omega)=\bar{\mathbf{u}}(x) & x\in\Gamma_1 \\ \left(\Gamma_1\cap\Gamma_2=\phi,\quad \Gamma_1\cup\Gamma_2=\partial\Omega\right) \end{cases} \tag{1}$$

where suppose that $a^{\varepsilon}_{ijhk}(x,\omega)$ (*i,j,h,k=1,…,n*) are the elastic coefficients of the random distribution with ε-size periodicity, and the jointing between joint surfaces and matrix are considered isotropic homogenous materials and continuous transition zones, so $\left\{a^{\varepsilon}_{ijhk}(x,\omega)\right\}$ is highly oscillating, but continuously varying.

Below SSOTS method will be discussed for the problem (1). Let $\xi=\dfrac{x}{\varepsilon}-\left[\dfrac{x}{\varepsilon}\right]\in Q^s$ denotes the local coordinates on 1-normalized cell of ε-cell $\varepsilon Q^s\subset\Omega$. Then $a^{\varepsilon}_{ijhk}\left(x,\omega\right)=a_{ijhk}\left(\xi,\omega\right)$ and $\mathbf{u}^{\varepsilon}(x,\omega)=\mathbf{u}(x,\xi,\omega)$. Inspired by the paper or books (Cui & Yang 1996 Oleinik et al. 1992 Jikov et al. 1994), by using constructive way following formulas on SSOTS solution of previous problem were obtained: The displacement solution of problem (1) can be expressed as follows

$$\begin{aligned} \mathbf{u}^{\varepsilon}\left(x,\omega\right)=\mathbf{u}^{0}\left(x\right)+\varepsilon\mathbf{N}_{\alpha_1}\left(\xi,\omega\right)\frac{\partial\mathbf{u}^{0}(x)}{\partial x_{\alpha_1}}+\varepsilon^{2}\mathbf{N}_{\alpha_1\alpha_2}\left(\xi,\omega\right)\frac{\partial^{2}\mathbf{u}^{0}(x)}{\partial x_{\alpha_1}\partial x_{\alpha_2}} \\ +\varepsilon^{3}\mathbf{P}_1(x,\xi,\omega)\quad x\in\Omega, \end{aligned} \tag{2}$$

where $\mathbf{u}^{0}(x)$ is the homogenization solution defined on global Ω, $\mathbf{N}_{\alpha_1}\left(\xi,\omega\right)$ and $\mathbf{N}_{\alpha_1\alpha_2}\left(\xi,\omega\right)$ $\left(\alpha_1,\alpha_2=1,\cdots,n\right)$ are n-order matrix-valued functions defined on 1-normalized Q, and they have following forms

$$\mathbf{N}_{\alpha_1}\left(\xi,\omega\right)=\begin{pmatrix} N_{\alpha_1 11}\left(\xi,\omega\right) & \cdots & N_{\alpha_1 1n}\left(\xi,\omega\right) \\ \vdots & \cdots & \vdots \\ N_{\alpha_1 n1}\left(\xi,\omega\right) & \cdots & N_{\alpha_1 nn}\left(\xi,\omega\right) \end{pmatrix} \tag{3}$$

$$\mathbf{N}_{\alpha_1\alpha_2}\left(\xi,\omega\right)=\begin{pmatrix} N_{\alpha_1\alpha_2 11}\left(\xi,\omega\right) & \cdots & N_{\alpha_1\alpha_2 1n}\left(\xi,\omega\right) \\ \vdots & \cdots & \vdots \\ N_{\alpha_1\alpha_2 n1}\left(\xi,\omega\right) & \cdots & N_{\alpha_1\alpha_2 nn}\left(\xi,\omega\right) \end{pmatrix} \tag{4}$$

And $\mathbf{N}_{\alpha_1}(\xi,\omega)$, $\mathbf{N}_{\alpha_1\alpha_2}(\xi,\omega)$ $(\alpha_1,\alpha_2=1,\cdots,n)$ and $\mathbf{u}^0(x)$ are determined in following ways:

1) For any sample ω^s, $\mathbf{N}_{\alpha_1 m}(\xi,\omega^s)$ $(\alpha_1,m=1,\cdots,n)$ are the solutions of following problems

$$\begin{cases} \dfrac{\partial}{\partial\xi_j}\left[a_{ijhk}(\xi,\omega^s)\dfrac{1}{2}\left(\dfrac{\partial N_{\alpha_1 hm}(\xi,\omega^s)}{\partial\xi_k}+\dfrac{\partial N_{\alpha_1 km}(\xi,\omega^s)}{\partial\xi_h}\right)\right]=-\dfrac{\partial a_{ij\alpha_1 m}(\xi,\omega^s)}{\partial\xi_j} & \xi\in Q^s \\ \mathbf{N}_{\alpha_1 m}(\xi,\omega^s)=0 & \xi\in\partial Q^s \end{cases} \tag{5}$$

2) From $\mathbf{N}_{\alpha_1 m}(\xi,\omega^s)$, the homogenization elasticity parameters $\{\hat{a}_{ijhk}(\omega^s)\}$ corresponding to the sample ω^s are calculated in following formula

$$\hat{a}_{ijhk}(\omega^s)=\int_{Q^s}\left(a_{ijhk}(\xi,\omega^s)+a_{ijpq}(\xi,\omega^s)\frac{1}{2}\left(\frac{\partial N_{hpk}(\xi,\omega^s)}{\partial\xi_q}+\frac{\partial N_{hqk}(\xi,\omega^s)}{\partial\xi_p}\right)\right)d\xi \tag{6}$$

3) One can evaluate the expected homogenized coefficients $\{\widehat{a}_{ijhk}\}$ in following formula

$$\widehat{a}_{ijhk}=\frac{\sum_{s=1}^{M}\hat{a}_{ijhk}(\omega^s)}{M},\ M\to+\infty \tag{7}$$

4) For any sample ω^s, $\mathbf{N}_{\alpha_1\alpha_2 m}(\xi,\omega^s)$ $(\alpha_1,\alpha_2,m=1,\cdots,n)$ are the solutions of following problems

$$\begin{cases} \dfrac{\partial}{\partial\xi_j}\left[a_{ijhk}(\xi,\omega^s)\dfrac{1}{2}\left(\dfrac{\partial N_{\alpha_1\alpha_2 hm}(\xi,\omega^s)}{\partial\xi_k}+\dfrac{\partial N_{\alpha_1\alpha_2 km}(\xi,\omega^s)}{\partial\xi_h}\right)\right]=\hat{a}_{i\alpha_2 m\alpha_1} & \\ \qquad -a_{i\alpha_2 m\alpha_1}(\xi,\omega^s)-a_{i\alpha_2 hk}(\xi,\omega^s)\dfrac{\partial N_{\alpha_1 hm}(\xi,\omega^s)}{\partial\xi_k} & \xi\in Q^s \\ \qquad -\dfrac{\partial}{\partial\xi_j}\left(a_{ijh\alpha_2}(\xi,\omega^s)N_{\alpha_1 hm}(\xi,\omega^s)\right) & \\ \mathbf{N}_{\alpha_1\alpha_2 m}(\xi,\omega^s)=0 & \xi\in\partial Q^s \end{cases} \tag{8}$$

5) $\mathbf{u}^0(x)$ is the solution of the homogenization problem with the homogenized parameters $\{\widehat{a}_{ijhk}\}$ defined on global Ω

$$\begin{cases} \dfrac{\partial}{\partial x_j}\left[\widehat{a}_{ijhk}\dfrac{1}{2}\dfrac{\partial}{\partial x_j}\left(\dfrac{\partial u_h^0(x)}{\partial x_k}+\dfrac{\partial u_k^0(x)}{\partial x_h}\right)\right]=f_i(x), & x\in\Omega \\ \mathbf{u}^0(x)=\overline{\mathbf{u}}(x), & x\in\Gamma_1 \\ \sigma_i(x)=\nu_j\widehat{a}_{jihk}\dfrac{1}{2}\left(\dfrac{\partial u_h^0(x)}{\partial x_k}+\dfrac{\partial u_k^0(x)}{\partial x_h}\right)=p_i, & x\in\Gamma_2 \\ \left(\Gamma_1\bigcap\Gamma_2=\phi,\ \Gamma_1\bigcup\Gamma_2=\partial\Omega\right) & \end{cases} \tag{9}$$

6) The strains can be evaluated approximately in following formulas:

$$\begin{aligned}\varepsilon_{hk}\left(\frac{x}{\varepsilon},\omega\right) &= \frac{1}{2}\left(\frac{\partial u_h^0(x)}{\partial x_k}+\frac{\partial u_k^0(x)}{\partial x_h}\right)\\ &+\sum_{l=1}^{2}\varepsilon^l\sum_{<\boldsymbol{\alpha}>=l}\frac{1}{2}\left[N_{\boldsymbol{\alpha}hm}\left(\frac{x}{\varepsilon},\omega\right)D_{\boldsymbol{\alpha}k}^{l+1}u_m^0(x)+N_{\boldsymbol{\alpha}km}\left(\frac{x}{\varepsilon},\omega\right)D_{\boldsymbol{\alpha}h}^{l+1}u_m^0(x)\right]\\ &+\sum_{l=1}^{2}\varepsilon^{l-1}\sum_{<\boldsymbol{\alpha}>=l}\frac{1}{2}\left[\frac{\partial N_{\boldsymbol{\alpha}hm}}{\partial \xi_k}+\frac{\partial N_{\boldsymbol{\alpha}km}}{\partial \xi_h}\right]\left(\frac{x}{\varepsilon},\omega\right)D_{\boldsymbol{\alpha}}^{l}u_m^0(x)\end{aligned} \tag{10}$$

where $\alpha=(\alpha_1,\alpha_2,\cdots,\alpha_l)$, $D_\alpha^l u_m^0(x)=\dfrac{\partial^l u_m^0(x)}{\partial x_{\alpha_1}\partial x_{\alpha_2}\cdots\partial x_{\alpha_l}}$. And then the stress tensor can be calculated in following formulae

$$\sigma_{ij}(x,\omega)=a_{ijhk}\left(\frac{x}{\varepsilon},\omega\right)\varepsilon_{hk}\left(\frac{x}{\varepsilon},\omega\right) \tag{11}$$

3.2 *Computation of the strength*

As the strain and stress tensor anywhere inside the investigated structure are obtained, the elasticity limit strength for the structure made from rock mass can be evaluated. Until now there is no strength criterion for the structure of rock mass with lots of random joints or/and cracks. In this paper, we employ the strength criterions on homogenous materials and the status of joint or/and crack expansions to define the elasticity limit strength of the structure of rock mass.

It's worthy to note that the employed strength criterion should be different for the different status of intact rock and jointing, such as tension and pression, the maximum principal stress theory should be cited for rock mass. In our computation, only the formulation of maximum stress criterion is shown. The formulas of other strength criterions can be easily found in textbook of solid mechanics or mechanics of rock mass.

The maximum principal stress theory assumes that failure occurs when the maximum principal stress σ_1 in the complex stress system equals to that at the yield point in the tensile test, where σ_1, σ_2 and σ_3 are the three principal stresses under the three dimensional complex stress states.

For a sample ω^s, all of strains and stresses inside any ε-cell belonging to the structure can be obtained through the formulas presented previously. Then, the strength $S(\omega^s)$ of the structure with random joint or/and crack distribution is obtained as the elasticity limit criterions is reached at some point for the sample ω^s. Thus to repeat previous calculation so many times, from Kolmogorov strong law of the large number, it follows that the expected strength $\hat{S}$ can be evaluated by following formula:

$$\hat{S}=\frac{\sum_{s=1}^{M}S(\omega^s)}{M} \tag{12}$$

However, the expected strength $\hat{S}$ can not totally represent the strength properties of the structure of random joint/crack distribution. The yield of some location may lead to the collapse of the whole structure. Therefore, the minimal strength of the structure of random joint/crack distribution is sometime worthier than the expected one for the design of rock mass structures. The minimal strength can be defined as following formula:

$$S_{\min} = \min_{s=1,\cdots,M} \left\{ S(\omega^s) \right\} \tag{13}$$

4 THE PROCEDURE OF MSA COMPUTATION BASED ON SSOTS

Based on the multi-scale representation of the rock mass with random joint/crack distribution in sections 2 and the SSOTS formulation in section 3, the algorithm procedure of predicting the mechanical parameters of structure with random joint/crack distribution is following:

step 1. Generate a distribution model P of joints or/and cracks based on the statistical characteristics of the random joint or/and crack distribution, and determine the material coefficients $\left\{ a_{ijhk}\left(\frac{x}{\varepsilon},\omega^s\right) \right\}$ on $\varepsilon Q(\varepsilon)$ as follows:

$$a_{ijhk}\left(\frac{x}{\varepsilon},\omega^s\right) = \begin{cases} a_{ijhk}, & x \in \varepsilon \widehat{Q}(\varepsilon) \\ a'_{ijhk}, & x \in \varepsilon \tilde{Q}(\varepsilon) \end{cases}, \ \omega^{\mathrm{s}} \in P,$$

where $\varepsilon \widehat{Q}(\varepsilon)$ is the domain of intact rock and $\varepsilon \tilde{Q}(\varepsilon)$ the domain of joints and jointing in $\varepsilon Q(\varepsilon)$ and $\left\{ a_{ijhk} \right\}$ and $\left\{ a'_{ijhk} \right\}$ are the material coefficients of them, respectively.

step 2. Evaluate FE solution $\mathbf{N}^h_{\alpha_1 m}\left(\xi,\omega^s\right)$ $\left(\alpha_1, m = 1,\cdots,n\right)$ of $\mathbf{N}_{\alpha_1 m}\left(\xi,\omega^s\right)$ by solving problem (5) for $\omega^s \in P$. Then the sample homogenization coefficients $\left\{ \hat{a}^r_{ijhk}(\omega^s) \right\}$ can be calculated through formula (6). And then to evaluate FE solution $\mathbf{N}^h_{\alpha_1\alpha_2 m}\left(\xi,\omega^s\right)$ of $\mathbf{N}_{\alpha_1\alpha_2 m}\left(\xi,\omega^s\right)$ $\left(\alpha_1, m = 1,\cdots,n\right)$ for $\omega^s \in P$ by solving problem (7).

step 3. For $\omega^s \in P$, $s = 1,2,...,M$, step 1 to 2 are repeated M times. Then M sample homogenization coefficients $\left\{ \hat{a}_{ijkh}(\omega^s) \right\}$ are obtained. The expected homogenization coefficients $\left\{ \widehat{a}_{ijkh} \right\}$ for the rock mass with random joint or/and crack distribution can be evaluated in formulae (7).

step 4. The homogenization solution $\mathbf{u}^0(x)$ can be obtained by solving homogenization problem (9) with the homogenization coefficients $\left\{ \widehat{a}_{ijkh} \right\}$. For some typical structures/components, $\mathbf{u}^0(x)$ can be exactly obtained from solid mechanics.

step 5. For the sample ω^s, evaluate the stain fields anywhere inside the investigated structure by $\mathbf{N}^h_{\alpha_1 m}\left(\frac{x}{\varepsilon},\omega^s\right)$, $\mathbf{N}^h_{\alpha_1\alpha_2 m}\left(\frac{x}{\varepsilon},\omega^s\right)$ $\left(\alpha_1, m = 1,\cdots,n\right)$, and $\mathbf{u}^0(x)$ through formulas (10) in section 3. The stresses can be calculated through Hooke's Law (11).

step 6. By using the strength S_m of intact rock, the strength S_P of jointing and the criterion of joint or/and crack expansion, the elasticity limit load of the structure for ω^s can be determined by using iteration procedure. After that, the strength limit of the structure for ω^s, denoted by $S(\omega^s)$, is calculated according to the critical load and the homogenization stiffness parameters $\left\{\hat{a}_{ijkh}(\omega^s)\right\}$.

step 7. For $\omega^s \in P$, $s = 1, 2, ..., M$, step 5 to 6 are repeated. Then M sample strengths $S(\omega^s)$ are obtained. The expected strength $\widehat{S}$ and the minimal strength $S_{\min}$ for the structure with random joint/crack distribution can be evaluated in formulae (12) and (13).

If there are so plenty of random joints or/and cracks inside structure and the differences of their sizes are very large. One should divide all of random joints or/and cracks into several classes according to their size. They are divided into 4 classes, N=4, shown in Figure 1. As $\left\{\widehat{a}^r_{ijhk}\right\}$, $\widehat{S}^r$ and $S^r_{\min}$ (r=4) are obtained, they are used as the elastic coefficients and strength of new intact rock in the next cycle with *r=N-1*, i.e. if it's not the first cycle ($r \neq N$), the material coefficient of the intact rock is the homogenized coefficient $\left\{\widehat{a}^{r+1}_{ijhk}\right\}$, and the elasticity limit strength of the intact rock are the strength $\widehat{S}^{r+1}$ and $S^r_{\min}$, respectively, which are evaluated in former cycle with (r+1) class.

As the last cycle $r = 1$ is completed, the expected homogenization coefficients $\left\{\widehat{a}^1_{ijhk}\right\}$ and expected elasticity limit strength $\widehat{S}^1$ and $S^1_{\min}$ are obtained. And then $\widehat{S}^1$ and $S^1_{\min}$ are defined as the effective elastic coefficients and expected / minimal strength of the investigated structure/component made from the rock mass with random distribution of multi-scale joints or/and cracks.

5 NUMERICAL EXPERIMENT

To verify the previous algorithm, the homogenized coefficients of the rock mass are evaluated. Three models of random joint distribution in 2-D case are considered in three examples, respectively.

In every example, the joints are divided into four classes G_1, G_2, G_3 and G_4 by the length of joint trace, the length of whose statistic screen is denoted by ε^1, ε^2, ε^3 and ε^4, respectively, and the length of joints in every class is supposed to be uniform distribution in a certain interval [a,b], shown in Table 1, and there are 4 joints in every ε^i-screen, and the thickness of the jointing in every joint is 1% of its length.

Table 1. The screen size and interval of each class

	Statistics screen scale	Intervals of each group
G_1	ε^4=1m	[0.1-0.25]
G_2	ε^3=5m	[0.5-1.0]
G_3	ε^2=15m	[2.5-5.0]
G_4	ε^1=20m	[7.5-10]

And in each example, the material coefficients of intact rock and jointing are supposed to be same, shown in Table 2.

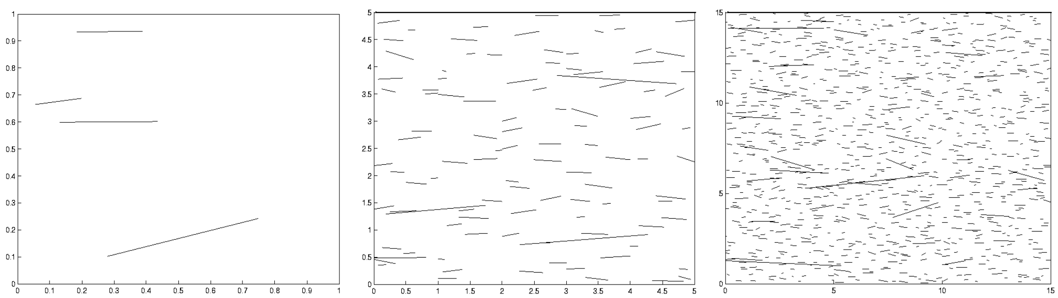
For the first example, the inclination of the joints in each class is supposed to be uniform distribution between 0^o and 360^o, denoted by UD (0^o, 360^o). For the second example, the inclination of the joints in each class is supposed to be normal distribution with expectation 0^o and mean square deviation 10^o, denoted by ND (0^o, 10^o). For the last example, the inclination of the joints in each group is supposed to be normal distribution with expectation 50^o and mean square deviation 10^o, denoted by ND (50^o, 10^o).

Table 2. The material coefficients of intact rock and jointing

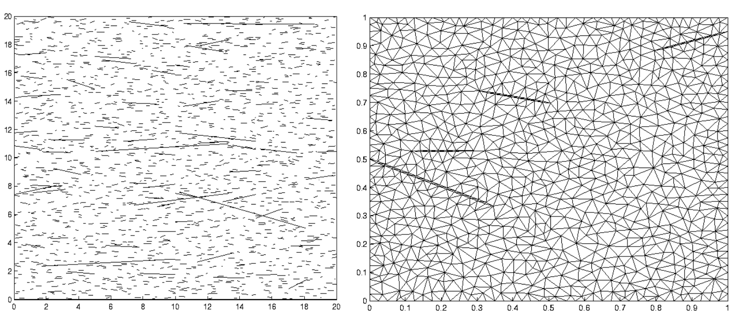
Intact rock	Jointing
$\begin{pmatrix} 3.333E4 & 8.3333E3 & 0 \\ 8.3333E3 & 3.3333E4 & 0 \\ 0 & 0 & 1.25E4 \end{pmatrix}$	$\begin{pmatrix} 3.3333E2 & 8.3333E1 & 0 \\ 8.3333E1 & 3.3333E2 & 0 \\ 0 & 0 & 1.25E2 \end{pmatrix}$

Table 3. The expected homogenized results of each scale screen for UD (0^o, 360^o)

$\varepsilon^4 \begin{pmatrix} 3.0086\ E4 & 7.064\ E3 & 0 \\ 7.064\ E3 & 2.9810\ .E4 & 0 \\ 0 & 0 & 1.1441\ E4 \end{pmatrix}$	$\varepsilon^3 \begin{pmatrix} 2.7614E4 & 6.272E3 & 0 \\ 6.272E3 & 2.7708E4 & 0 \\ 0 & 0 & 1.0720E4 \end{pmatrix}$
$\varepsilon^2 \begin{pmatrix} 2.5695\ E4 & 5.528\ E3 & 0 \\ 5.528\ E3 & 2.4810\ E4 & 0 \\ 0 & 0 & 9.833\ E3 \end{pmatrix}$	$\varepsilon^1 \begin{pmatrix} 2.2026E4 & 4.623E3 & 0 \\ 4.623E3 & 2.1843E4 & 0 \\ 0 & 0 & 8.666E3 \end{pmatrix}$



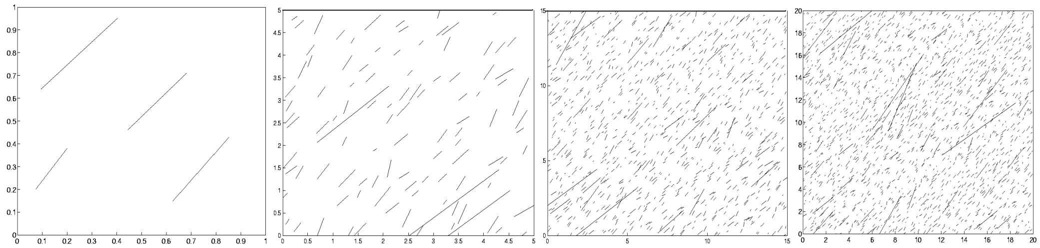
a. Joints in ε^4-screen b. Joints in ε^3-screen c. Joints in ε^2-screen



d. Joints in ε^1-screen e. Mesh partition

Figure 2. The statistical model of joints for ND(0^o,10^o) and mesh partition

By virtue of above specified data and the simulation method of the joints, the joints in each screen can be easily generated for one sample. In order to show clearly the distribution of the joints in the rock mass, the joints in ε^i-screen are generated as well the joints in the screen smaller than ε^i together. The distribution model of joints for one sample of UD (0^o, 360^o), ND (0^o, 10^o) and ND (50^o, 10^o) is shown in Figure 1, Figure 2 and Figure 3, respectively.



a. Joints in ε^4-screen b. Joints in ε^3-screen c. Joints in ε^2-screen d. Joints in ε^1-screen

Figure 3. The joints statistical model for ND (50^o,10^o)

Table 4 The expected homogenized results of each scale screen for ND (0^o, 10^o)

ε^4	$\begin{pmatrix} 3.2589E4 & 6.868E3 & 0 \\ 6.868E3 & 2.7364E4 & 0 \\ 0 & 0 & 1.1186E4 \end{pmatrix}$	ε^3	$\begin{pmatrix} 3.1960E4 & 5.815E3 & 0 \\ 5.815E3 & 2.3089E4 & 0 \\ 0 & 0 & 1.0148E4 \end{pmatrix}$
ε^2	$\begin{pmatrix} 3.1271E4 & 4.853E3 & 0 \\ 4.853E3 & 1.9136E4 & 0 \\ 0 & 0 & 9.072E3 \end{pmatrix}$	ε^1	$\begin{pmatrix} 3.0351E4 & 3.846E3 & 0 \\ 3.846E3 & 2.3089E4 & 0 \\ 0 & 0 & 1.0148E4 \end{pmatrix}$

Table 5 Final result

UD	$\begin{pmatrix} 2.2026E4 & 4.623E3 & 0 \\ 4.623E3 & 2.1843E4 & 0 \\ 0 & 0 & 8.666E3 \end{pmatrix}$
ND(0o,10o)	$\begin{pmatrix} 3.0351E4 & 3.846E3 & 0 \\ 3.846E3 & 1.4995E4 & 0 \\ 0 & 0 & 7.755E3 \end{pmatrix}$
ND(50o,10)	$\begin{pmatrix} 2.1005E4 & 8.194E3 & 2.094E3 \\ 8.194E3 & 2.1873E4 & 2.592E3 \\ 2.094E3 & 2.592E3 & 9.810E3 \end{pmatrix}$

The 50 distribution samples of the joints in each screen for every example are sampled. Every sample with joints is partitioned as shown in Figure 2.e. And the expected homogenized coefficients can be calculated by the procedure given in section 4. The detailed results of UD and ND(0^o,10^o) are given in Table 3 and Table 4, respectively. The detailed results of ND (50^o,10^o) are omitted owing to the limitation of space. The final expected homogenized results for UD (50^o,10^o), ND (0^o,10^o) and ND (50^o,10^o) are given in Table 5.

By using SMS method in this paper the elasticity limit strengths of the rock mass with random joints/cracks distribution, including tension and compression, bending and twist, have been calculated, and the numerical results on expected elasticity strength and minimal elasticity strength were obtained. For the space limitation of this paper those on rock mass strength are omitted here.

6 CONCLUSIONS

In this paper one kind of structures of rock mass with plenty of joints or/and cracks is considered, they are defined as the structures of the materials with random distribution of multi-scale joints or/and cracks. And the micro-structure of rock mass with plenty of multi-scale joints or/and cracks is represented.

A new statistically second-order two-scale methods for the predicting the mechanics performances of them is presented, including the second-order two-scale asymptotic expression on the displacement vector, the formulations of the expected homogenization constitutive parameters, elasticity limit strength, and the algorithm procedures.

For some different random distribution models the expected homogenization constitutive parameters are predicted by SSOTS method. And the numerical experiments show that the micro-behaviors inside the structure with plenty of joints or /and cracks can be captured exactly by SSOTS method. And all of numerical results show that SSOTS method is valid and available.

REFERENCES

Cui J.Z. & Shan Y.J. 2000. The two-scale analysis algorithms for the structure with several configurations of small periodicity. In Topping B.H.V. (ed.), *Computational Techniques for Materials, Composites and Composite Structures*, CIVIL-COMP PRESS.

Cui J.Z. & Shih T.M. & Wang Y.L. 1997. The two-scale analysis method for bodies with small periodic configuration. *Structural Engineering and Mechanics* 7(6): 601-614, *Invited Paper in CASCM-97, Sydney, Australia, Proc. of CASCM-97*.

Cui J.Z. & Yang H.Y. 1996. A dual coupled method of boundary value problems of PDE with coefficients of small period. *International Journal of Computer Mathematics* 14: 159-174.

Jikov V.V. & Kozlov S.M. & Oleinik O.A. 1994. *Homogenization of differential operators and integral functions.* Berlin: Springer

Li Y.Y. & Cui J.Z. 2004. Two-scale analysis method for predicting heat transfer performance of composite materials with random grain distribution. *Science in China Series A: Mathematics*, 47: 101-110.

Li Y.Y. & Cui J.Z. 2005. The multi-scale computational method for mechanics parameters of composite materials with random grain distribution. *Journal of Composites Science & Technology* 65: 1447-1458.

Oleinik O.A. & Shamaev A.S. & Yosifian G.A. 1992. *Mathematical Problems in Elasticity and Homogenization.* Amsterdam: North–Holland.

Shan Y.J. & Cui J.Z. & Liang F.G. 2002. Expected Slide Path Method For Stability Analysis of Rock Mass Based on Statistics Model of Joints and Stress Field. *Chinese Journal of Rock Mechanics and Engineering*. 21(2): 151-157

Yu Y. & Cui J.Z. & Han F. 2008. An effective computer generation method for the composites with random distribution of large numbers of heterogeneous grains. *Composites Science and Technology* 68: 2543-2550.